Balancing of Rotating and Reciprocating Systems in Engine: Basic Understanding (Part I)

The main objective of this tutorial is to introduce basic requirements and concepts of balancing piston/connecting-rod/crankshaft assembly in an engine. In order to be able to portray a coherent picture we need to introduce some simple tools used in statics and dynamics of objects. The approach taken here is combination of algebraic and geometric presentation to maximize physical understanding. But first, a review of some basic laws is in order. A three-dimensional system or object is in static equilibrium if summation of all external applied forces and moments in each of the three perpendicular X, Y, and Z directions are zero. Also, according to Newton's dynamic law, summation of all external applied forces in each of the X, Y, and Z directions must be equal to the mass of the object multiplied by the X, Y, and Z components of acceleration of the center of mass, receptively.

In dynamic analysis of systems, it is customary and convenient to convert a complex system to a more simplified version or model which is dynamically equivalent. Consider a round disk with an imbalance mass of "m" due to inaccuracies in design or manufacturing. Point G is the center of mass of the combined system of disk with mass of M and the mass m. Assuming perfect rigidity, the entire real system can be modeled as a mass of (M+m) located at the point G connected by a massless connecting rod to the bearing, as shown in Fig. 1. Dynamic analysis of the real system in Fig. 1 then proceeds equivalently as shown in Fig. 2 by finding the forces that are transferred to the bearing. In Fig. 2, the so-called method of free-body-diagram is used. In this analysis we will not concern ourselves with such issues as torsional and rotational resonances and are only after the basic requirements of balancing.

For rotational system, we speak of static and dynamic balancing of a system. Imagine a rotating system that has an imbalance mass m_2 at a distance of r_2 from the center of rotation. If this object is laid on a horizontal knife-edge type support free to move, it will rotate so as the imbalance mass m_2 locates itself at the lowermost position due to gravity effects. Technically, it will rotate until the mass m_2 is below the center of mass of the object. Note that in this object, having the imbalance mass of m_2, the geometric center and mass center (i.e. center of mass) are not the same. By static balancing, we are trying to make these two centers the same. In this simple object, it is done by adding an extra mass m_1 at the radius r_1 such that the two centers coincide. Mathematically this means that $m_1 r_1 = m_2 r_2$ in order to satisfy the static equilibrium condition mentioned above. The question here is whether the system will now be balanced dynamically ? To answer this question we refer to Fig. 3. Here, the same object that was statically balanced in Fig. 4 is arranged with two side bearings and rotated at a constant angular speed of ω. For dynamic balance we need, first, that $F_1 = F_2$ or $m_1 r_1 \omega^2 = m_2 r_2 \omega^2$. This leads to the same requirement for the static balancing above, that is $m_1 r_1 = m_2 r_2$. Hence, as far as the inertia forces are concerned the system is balanced. But, second, since these two forces are not colinear, they produce a torque "c" which is equal to $F_1 a = F_2 a$. This couple (or torque or moment) rotates due to shaft rotation and is called vibrational torque or couple. Therefore, as it is, the system is not dynamically balanced. A major conclusion here is that if we have dynamic balance this means we do have a static balance as well; but the converse statement is not correct. One excellent example is balancing the automotive tire on dynamic balancer devices.

Perhaps an example makes it more clear. Figure 5 shows an example in which an imaginary shaft with two imbalance masses is to be balanced dynamically by addition of two masses in the planes identified as "0" and "3" at given and known distances of r_0 and r_3 from the axis of the rotation. We are after masses m_0 and m_3 and the angles at which they should be located for dynamic balance. Here, geometrical approach is selected although this can easily be done algebraically or automated by a computer program. Figure 6 shows details of the solution. In dynamics, any applied force can be conceptually transferred to any other location of the same object by addition (and application) of an appropriate moment. Dynamically speaking, no effects will be introduced by such an undertaking and this is usually used as a solution strategy. In Fig. 6 we have moved all forces due to eccentric masses to plane "0" as shown. The double-headed arrows indicate the applied moments and follow the so-called right-hand-thumb rule. The solution methodology starts with the couple or moment diagram to find the value and orientation of the mass m_3. From this moment diagram and knowing $c_3 = (m_3 r_3 \omega^2) a_3$, one can find the value of mass m_3. To find the m_0 we need to satisfy another condition of dynamic balancing which states that the vectorial sum of all the applied forces must be equal to zero. Figure 7 shows both the moment and force diagrams to satisfy the two conditions of the dynamic balancing. This completes our dynamic balancing of the example in Fig. 5. We will continue this tutorial in part II when we discuss dynamic balancing of reciprocating systems such as piston/connecting-rod/crank-shaft assembly for a single and a multi-cylinder engines.

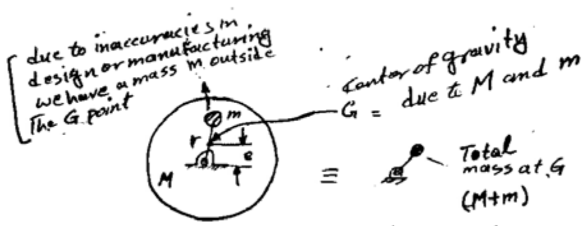


Figure 1. Rotating disk with mass M having an imbalance mass of m

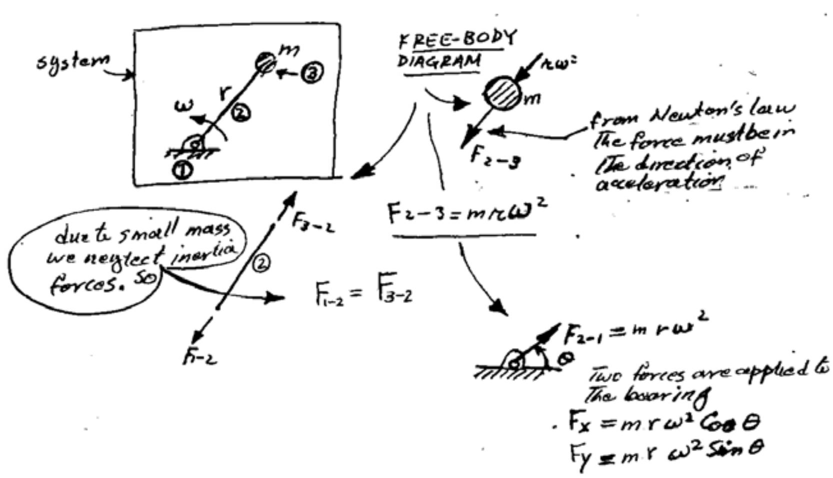


Figure 2. Dynamic equivallent of the real system shown in Fig. 1.

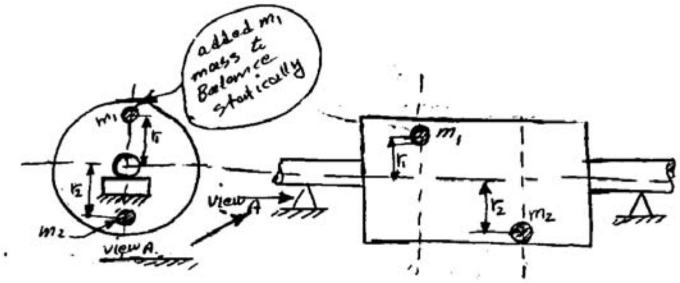


Figure 3. Static balancing of a rotating shaft.

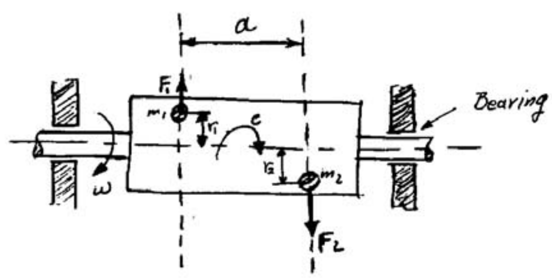


Figure 4. A statically-balanced system is not in dynamic balance.

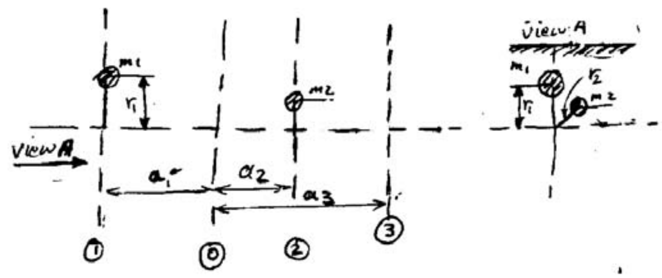


Figure 5. An example to balance a shaft dynamically.

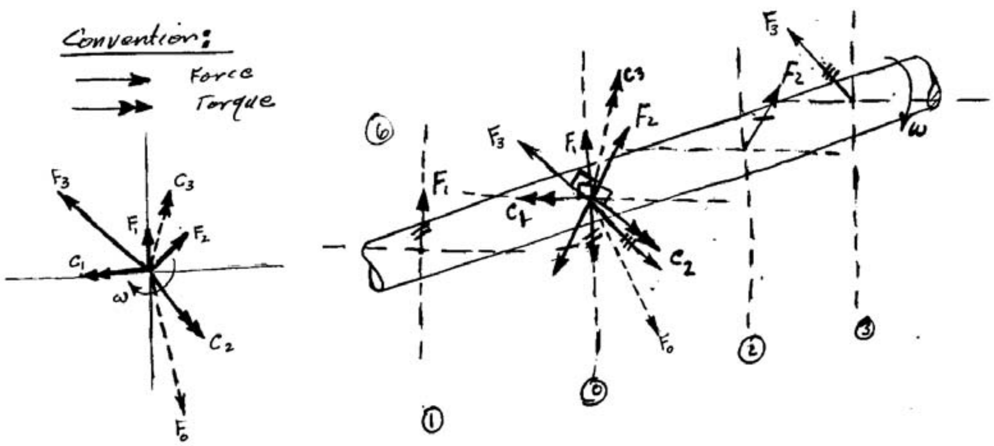
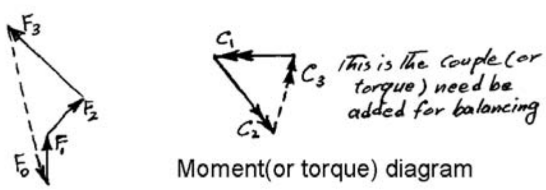


Figure 6. Geometric method of dynamic balancing the shaft in Fig. 5. Note that forces F1, F2, and F3 are translated to the zero plane and an appropriate moment (or torque) equal to the magnitude of the force times the displacement distance is added for each force as C1, C2, C3, respectively. The diagram to the left shows all the forces and moments applied to the system being equivallent to the original real system as far as the dynamic forces and moments are concerned.



Moment(or torque) diagram

Force diagram

Figure 7. Finding the unknowns by force and moment diagrams.

Balancing of Rotating and Reciprocating Systems in Engine: Basic Understanding (Part II)

In part I of this series basics of the static and dynamic balancing of rotating objects were discussed. In this part our focus is targeted at the reciprocating systems and in particular piston/connecting-rod/crank-shaft mechanism. However, before details discussion of this subject, we need to review some concepts in dynamics. Two systems of bodies are said to be dynamically equivalent if their motions are the same under the same set of forces and moments. This is illustrated in Fig. 1 where a real object is idealized by its dynamically equivalent system consisting of two masses m_1 and m_2 connected by a rigid and massless rod. The concept of the equivalent idealized system is very useful in solving complex problems. Mathematically we need to satisfy the following equations in order to ensure dynamic equivalency:

$m_1 + m_2 = m$ (1), Mass of ideal system = Mass of the real object
$m_1 a_1 - m_2 b_1 = 0$ (2), Identify location of the center of mass for the ideal object
$m_1 a_1^2 + m_2 b_1^2 = I_G$ (3), Moment of inertia of ideal system = Moment of inertia of real object

Where I_G is the moment of inertia of the real object about the center of mass and m_1 , m_2 , a_1 , and b_1 are the parameters of the idealized system to be determined by the above set of equations. However, we have four unknowns (and m_1 , m_2 , a_1 , and b_1), two knowns (m and I_G), and three equations. Hence, one of the unknowns must be arbitrarily assumed.

Consider a real (but frictionless and yet-unbalanced) reciprocating system that consists of a piston, connecting rod, and crank arm that is rotating at an angular velocity of ω. We also consider that the engine is not fired, hence only forces and moments due to the motion of the reciprocating system are in question here. Our approach in finding the unbalance forces and moments is to find the dynamic equivalent of the connecting rod and the crank arm in the first stage. Then we consider the forces and moments that arise as consequences of motions of these dynamic equivalences. Figure 2 shows the first stage of the process to find a dynamic equivalence for the original real reciprocating system. The points G_2 and G_3 are the locations of the center of mass for the real crank arm and connecting rod, respectively. The forces applied to the bearing A due to the arm 2 can readily be calculated as it only has rotational motion. However, situation for the object 3 is quite different because of its combined rotational and transitional motions. Using the above system of equations, one attempts to find the dynamic equivalent of the object 3 by selecting the distance "$a_1 = a$" and finding other unknowns (m_1, m_2 , and b_1) using the above system of equations. It is not difficult to see that in general the solution gives the location of the mass m_2 not at position B (i.e. b_1 is not necessarily equal to b). However, in practice b_1 is very close to b. Another way to approach this problem is to select positions of the two masses m_1 and m_2 to be at C and B, respectively, and then use equations (1) and (2) to find values of the m_1 and m_2. In this method, however, the equation (3) is only approximately satisfied ($m_1 a_1^2 + m_2 b_1^2 \cong I_G$). The ideal system is not then completely equivalent to the real connecting rod. As far as the inertia moments are concerned they are not equivalent between the idealized and the real systems but inertia forces are and this equivalency is of primary importance in a first-order approximate analysis. This same treatment should also be applied to the body 2 to find m_3 and m_4 and again it is found that the inertia moments of the real and the idealized systems are not equivalent. But this is not important for the crank shaft 2 as designers usually add "extra mass" (crank throw) to bring its center of mass to the rotation axis at A, see Fig. 2.

Figure 3 shows the ideal system for the real reciprocating system. Note that there is no need to position mass m_4 at location A because it does not move and the vibrational moments and forces are zero. Also, as indicated, the rotating (centrifugal) force F_B (= $(m_2 + m_3)R\omega^2$) can easily be neutralized by proper design of the crank throw. Therefore, the only forces of our concern are those as a result of the motion of an idealized mass equal to ($m_1 + m_P$) located at the piston position B. Here, m_P is the mass of the original real piston. To find these forces we need to calculate the acceleration of the point B and then multiply it by the ($m_1 + m_P$) according to the Newton's law.

We take the origin of the x-axis at the TDC and it is not difficult to show that :

$X = (L + R) - (R \cos\theta + L \cos\varphi)$ and $\sin\varphi = (1/n) \sin\theta$, where $n=L/R$.

From $\cos\varphi = \mathrm{SQRT}(1- \sin^2\varphi) = \mathrm{SQRT}(1- (1/n^2)\sin^2\theta) \cong 1 - 0.5(1/n^2)\sin^2\theta$ we have

$X \cong (L + R) - RCos\theta - L + L/2(1/n^2)Sin^2\theta$ taking this good approximate relationship as an equality, second time derivative of the X gives the acceleration of the point B as:

$f = R\omega^2(Cos\theta + 1/nCos2\theta)$. It is customary to define the primary reciprocating (F_{P1}) and secondary reciprocating (F_{P2}) forces as follows:

$F_{P1} = (m_1 + m_P)\, R\, \omega^2 Cos\theta$ and $F_{P2} = (m_1 + m_P)\, (R\, \omega^2 /n)\, Cos2\theta$.

Note that since in most designs n is greater than one the F_{P2} is of secondary importance.

The analysis shows that we have total of three forces, F_B , F_{P1} , and F_{P2} acting on the system. Note that the direction of the force F_B changes but those of others are in the x direction always. A simple computer program can give the x and y components of the resultant force (i.e. F_X and F_Y) at any angular position of the crank arm θ. However, for clarity and visual effects a geometrical procedure is described in Fig. 4 in which resultant force (the vector ov) at an arbitrary crank angle θ is shown through a construction of three circles with the radial separations as illustrated. Note that the resultant force is transmitted to the main bearing at point A. Systematic application of the procedure shown in Fig. 4 at all crank angles traces a curve such as case(a) shown in Fig. 5. To reduce these applied forces on the main bearing, in one approach, a mass equal to about $(m_2+m_3) + (1/2 \text{ to } 2/3)(m_1+m_P)$ is added at radius R extending the crank arm and at opposite the mass $(m_2+ m_3)$ shown in Fig. 3. Case (b) in Fig. 5 shows effects of addition of such a mass on the resultant force on the main bearing. Substantial reduction of this forces is achieved in both x and y directions. This completes basic methodology for balancing a single reciprocating mechanism. In part III we discuss balancing methodology for multicylinder engines.

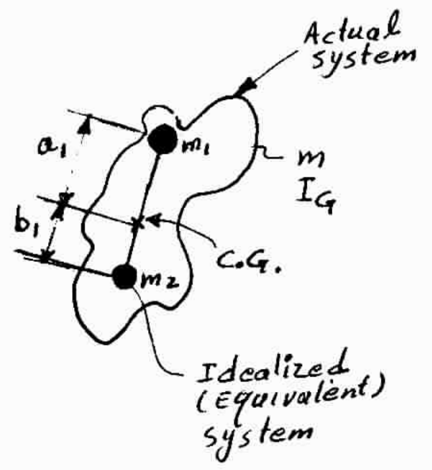


Figure 1. Dumbbell-shaped equivalent of an actual (real) object. Center of mass is the same for both.

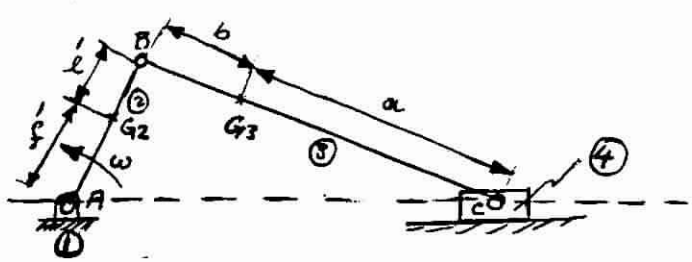


Figure 2. A reciprocating system. Linkages are idealized as shown. G2 and G3 are the center of masses for crank arm and connecting rod, respectively.

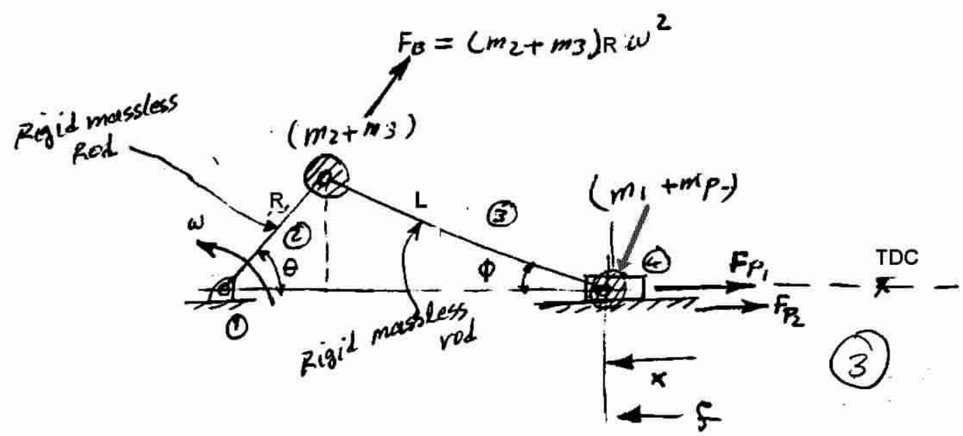


Figure 3. Dynamic equivalent of the real reciprocating system. Linkages are now are assumed to be rigid and massless.

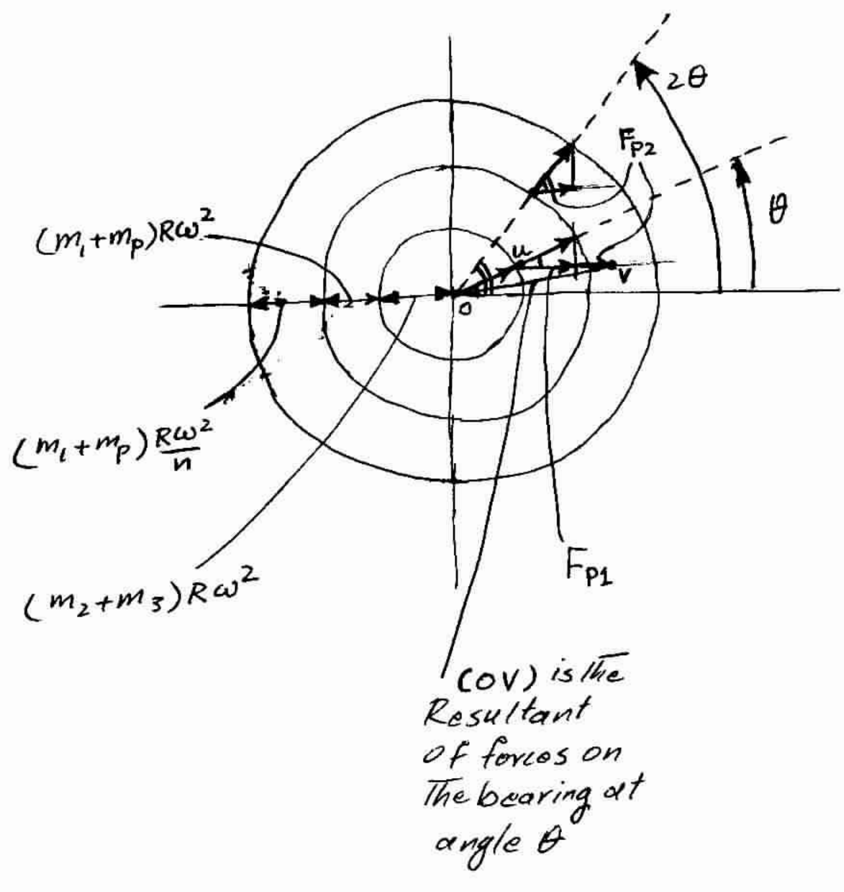


Figure 4. Shows geometric construction of the resultant force on the bearing as a result of the three forces discussed in the article.

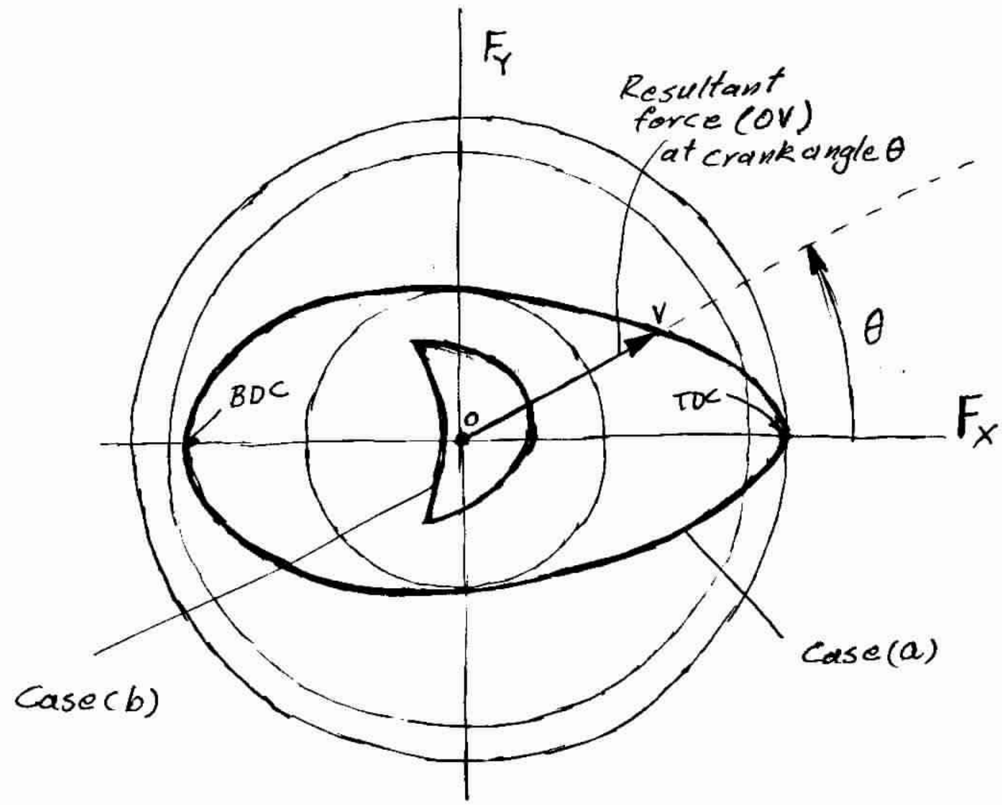


Figure 5. Polar plot of the resultant force on the bearing at different crankagles. Cases (a) and (b) are discussed in the article.

Balancing of Rotating and Reciprocating Systems in Engine: Basic Understanding (Part III)

In part I and II of this series static and dynamic balancing of rotating and reciprocating systems were discussed. For a reciprocating system, we learned that there were two reciprocating inertia forces. They were called primary and secondary forces in accordance to their importance. The secondary inertia force is lower in magnitude mainly because of the connecting rod length to crank arm ratio (L/R) being greater than a one. At this point, the reader is well equipped to understand the extension of what has been covered so far in part I and II to multicylinder engines. Here, as before, only the dynamic forces are considered and those imposed by the cylinder gas pressure are not considered.

In this tutorial, an example of a four cylinder engine is considered. However, for different cases the reader is referred to other sources. A methodology is needed for such a balancing exercise. Consider a general n' cylinder inline engine as shown in Fig. 1. As indicated in the part II, it is assumed that the rotating forces are balanced by a proper design of the crank throws. Consider the case where all pistons and distances between the neighboring ones are the same. We have seen in part II that the reciprocating inertia force for each piston-connecting-rod assembly is:

$$F = m_r R\omega^2 (\mathrm{Cos}\theta_1 + (1/n)\mathrm{Cos}2\theta_1)$$

Where n is the connecting rod to crank arm ratio (L/R) shown in Fig.2

and $m_r = m_1 + m_p$ is the effective reciprocating mass indicated in part II.

The force F_i (for the ith Cylinder) can be written as sum of primary and sec ondary forces:

$$F_i = m_r R\omega^2 \mathrm{Cos}\theta_i + m_r (R\omega^2/n)\mathrm{Cos}2\theta_i.$$

Considering that the above forces are all in vertical direction, the resultant force of all the reciprocating inertia forces applied to the bearing by all the piston-connecting-rod assemblies is:

A simple example can make the use of the last two equations clear. Let us consider a four cylinder engine with the crank arms as shown in Fig. 3. The firing order is shown in Table 1. These two equations suggest a systematic methodology shown in Table 2 to calculate the values for the summation terms.

$$\sum_{i=1}^{n'} F_i = \sum_{i=1}^{n'} m_r R\omega^2 \mathrm{Cos}(\theta_1 + \phi_i) + \sum_{i=1}^{n'} m_r (R\omega^2/n)\mathrm{Cos}(2\theta_1 + 2\phi_i),$$

θ_1 is the crank arm angular position of the first (i.e. the reference) piston and ϕ_is are as shown in the Fig.1. Considering a trigonometric relation,

$\mathrm{Cos}(\theta_1 + \phi_i) = \mathrm{Cos}\theta_1 \mathrm{Cos}\phi_i - \mathrm{Sin}\theta_1 \mathrm{Sin}\phi_i$, we have:

$$\sum_{i=1}^{n'} F_i = m_r R\omega^2 \mathrm{Cos}\theta_1 \sum_{i=1}^{n'} \mathrm{Cos}\phi_i - m_r R\omega^2 \mathrm{Sin}\theta_1 \sum_{i=1}^{n'} \mathrm{Sin}\phi_i + $$

$$+ m_r (R\omega^2/n)\mathrm{Cos}2\theta_1 \sum_{i=1}^{n'} \mathrm{Sin}2\phi_i - m_r (R\omega^2/n)\mathrm{Sins}2\theta_1 \sum_{i=1}^{n'} \mathrm{Sin}2\phi_i. \qquad (1)$$

Also, taking the cylinder number one as the reference for the moment arm calculation, the resul tan t moment of the above inertia forces is:

$$\sum_{i=1}^{n'} M_i = m_r R\omega^2 \mathrm{Cos}\theta_1 \sum_{i=1}^{n'} a_i \mathrm{Cos}\phi_i - m_r R\omega^2 \mathrm{Sin}\theta_1 \sum_{i=1}^{n'} a_i \mathrm{Sin}\phi_i + $$

$$+ m_r (R\omega^2/n)\mathrm{Cos}2\theta_1 \sum_{i=1}^{n'} a_i \mathrm{Sin}2\phi_i - m_r (R\omega^2/n)\mathrm{Sins}2\theta_1 \sum_{i=1}^{n'} a_i \mathrm{Sin}2\phi_i. \quad (2)$$

Table 2. Calculating the Σ (i.e. summation terms) equations (1) and (2).

ith cylinder	ϕ_i	$Cos\phi_i$	$Sin\phi_i$	$Cos2\phi_i$	$Sin2\phi_i$	a_i	$a_iCos\phi_i$	$a_iSin\phi_i$	$a_iCos2\phi_i$	$a_iSin2\phi_i$
1	**0**	**1**	**0**	**1**	**0**	**0**	**0**	**0**	**0**	**0**
2	**180**	**-1**	**0**	**1**	**0**	**a**	**(-1)a**	**0**	**a**	**0**
3	**180**	**-1**	**0**	**1**	**0**	**2a**	**(-2)a**	**0**	**2a**	**0**
4	**0**	**1**	**0**	**1**	**0**	**3a**	**(3)a**	**0**	**3a**	**0**
Summation		**0**	**0**	**4**	**0**		**0**	**0**	**6a**	**0**
		Primary reciprocating inertia forces are balanced		**secondary reciprocating inertia forces are not balanced**			**Primary vibrating moments are balanced**		**secondary vibrating moments are not balanced**	

It can be seen that both primary forces and moments, which are of prime importance, are balanced. As indicated before, the secondary unbalanced ones are of the lesser importance because of their division by the factor n. Note that n is usually larger than one.

Therefore, from equations (1) and (2) and the above table, the unbalanced inertia force and moment that are applied to the bearings are:

$$\sum_{i=1}^{n'} F_i = m_r R\omega^2\{[(Cos2\theta_1)/n](4)\} = \frac{(4)m_r R\omega^2}{n} Cos2\theta_1, \quad (3)$$

$$\sum_{i=1}^{n'} M_i = m_r R\omega^2\{[(Cos2\theta_1)/n](6a)\} = \frac{(6a)m_r R\omega^2}{n} Cos2\theta_1 \quad (4)$$

Extension of this basic solution methodology to a more complex geometry and cylinder arrangement is straightforward. Secondary unbalanced forces can be balanced by use of a twin countershaft design. These shafts are mounted each on one side of the engine and turn at twice the crankshaft rotation speed because of the $2\theta_1$ term in equation (3). Also, keep in mind that for the total force applied to the bearing, the gas pressure force on the piston should be algebraically added to the inertia forces. Generally, at low engine speeds the gas pressure forces dominate the inertia forces near the TDC in the power stroke. At high-speed however, they become comparable and the inertia forces can dominate depending on the engine speed. This completes demonstration of the balancing methodology applied to a simple four-cylinder engine arrangement. For more details, the reader invited to refer to Vehicle and Engine Technology by H. Heisler as a good starting point.

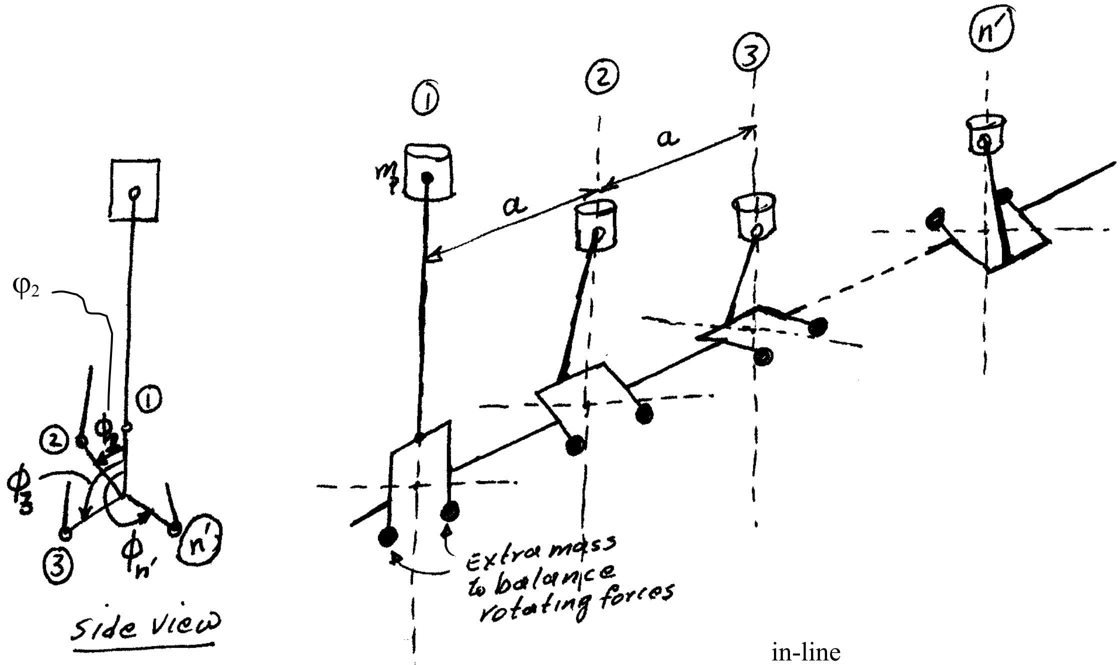


Figure 1: A general in-line n′-cylinder engine. ϕ_2 is the angle between the crank arms of the cylinder 1 and Cylinder 2 and likewise for others as shown. Note that $\phi_1 = 0$

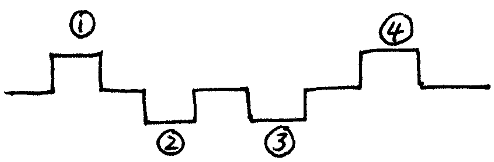


Figure 3. Crankarm arrangement for the example given

Cylinder Number

θ	1	3	4	2
0–180	P	C	I	E
180–360	E	P	C	I
360–540	I	E	P	C
540–720	C	I	E	P

P: Power stroke
E: Exhaust stroke
C: Compression stroke
I: Intake stroke

Table 1. Firing order

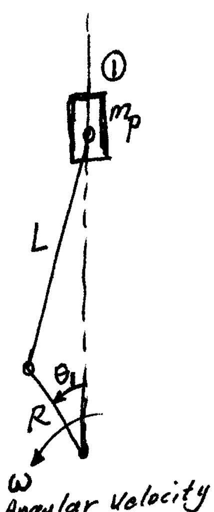


Figure 2. A single reciprocating system